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REVIEW

Turbulence and Stochastic Processes: Kolmogorov's Ideas 50 Years On. Edited by J. C. R. HUNT, O. M. PHILLIPS and D. WILLIAMS. *Proceedings of the Royal Society, London* A, vol. 434, 1991, pp. 1–240. £19.50.

This collection of papers commemorates the fiftieth anniversary of the publication of Kolmogorov's seminal paper 'The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers' (Kolmogorov 1941 *a*). In this five-page paper, Kolmogorov introduced first the concept of 'local isotropy' in which attention is focused on velocity differences $\Delta u = u(x+r)-u(x)$, and second his similarity hypotheses which identify the mean rate of dissipation of energy $\bar{\epsilon}$ as the key dimensional quantity which (in conjunction with kinematic viscosity ν) determines the small-scale statistical properties of the turbulence. It followed immediately that, when r is in the inertial range of lengthscales (where ν was assumed to play no part),

$$\overline{(\Delta u)^2} = C(\bar{\epsilon} r)^{2/3}, \qquad (1)$$

where C is a universal constant. This celebrated *Kolmogorov law* has served as one of the very few fixed points of turbulence theory and research over the past fifty years. The paper has had an extraordinary, albeit indirect, influence on the development of the subject over this half-century, and it is fitting that the Royal Society, of which Kolmogorov was himself a Foreign Member, should have published this commemorative special issue.

The volume starts with a reprinting of the English translations (by V. Levin) of the above paper and of the subsequent paper 'Dissipation of energy in the locally isotropic turbulence' (Kolmogorov 1941 *b*) in which Kolmogorov obtained the result

$$\overline{(\Delta u)^3} = -\tfrac{4}{5}\bar{\epsilon} r \qquad (2)$$

(again for r in the inertial range). Levin's translation is far from perfect, and one may regret that the Editors did not seize this opportunity to provide a new, more accurate translation, which would not so effectively obfuscate the nuances of Kolmogorov's argument. They have instead limited themselves to certain emendations, a dangerous procedure, since first these are incomplete (e.g. $\overline{u_\alpha^2}$ is printed as $\bar{u}_\alpha^2$, an error present in Levin's translation, but not in the original Russian version of Kolmogorov), and second, certain obscurities have actually been *introduced* in the course of 'emendation' (e.g. Kolmogorov's footnote "All results of §3 are quite similar to that obtained in (1), (2) and (4) for the case of isotropic turbulence in the sense of Taylor" refers in the original to References [1], [2] and [4]; but the references are now listed alphabetically without numbers, so that the footnote, faithfully reproduced, becomes extremely puzzling, to say the least).

The remaining papers in the volume are written by V. I. Arnol'd; R. Temam; D. Williams; R. H. Kraichnan; B. B. Mandelbrot; U. Frisch; Z.-S. She, E. Jackson & S. A. Orszag; O. M. Phillips; C. Van Atta; C. H. Gibson; K. R. Sreenivasan; J. C. R. Hunt & J. C. Vassilicos; D. B. Spalding; and K. N. C. Bray & R. S. Cant. Arnol'd's paper provides an informal glimpse of Kolmogorov's preoccupations in relation to dynamical systems and the problem of transition to turbulence as represented in his 1958/59 seminar programme at Moscow University. Arnol'd presents 'the Kolmogorov system' in the form

$$\dot{x}_i = \sum a^i_{jk} x_j x_k - b_i x_i + f_i,$$

which observes no known notational convention – it is hard to see how this could pass any vigilant referee. A more spectacular misprint occurs in the paper by Frisch who, in considering a quantity ϵ_i (the mean dissipation rate in a sub-ensemble labelled *i*) writes (p. 92): "From (6) and (7), we obtain

$$\frac{1}{N}\sum_i (\epsilon_i)^{\frac{1}{3}p} = \frac{1}{N}\sum_i (\epsilon_i)^{\frac{1}{3}p}.$$

This relation is contradictory, except for $p = 3$." Really?

The papers are of three types: (*a*) those that treat mathematical or probabilistic issues having some bearing on the problem of turbulence; (*b*) those that consider applications of the Kolmogorov theory in such areas as oceanography, interstellar gas dynamics, and combustion; and (*c*) those that focus on the problem of intermittency of the dissipation field $\epsilon(\boldsymbol{x}, t)$, a problem which, as recognized by Kolmogorov himself (Kolmogorov 1962) undermines and invalidates the hypotheses presented in his first 1941 paper. These hypotheses imply not only the results (1) and (2) but also the more general result

$$\overline{(\Delta u)^p} = C_p(\bar{\epsilon}r)^{p/3} \tag{3}$$

for arbitrary *p*. Experimental observations summarized by Anselmet *et al.* (1984) indicate increasing departures from the law (3) as the exponent *p* increases. In his 1962 paper, following Oboukhov (1962), Kolmogorov attempted to deal with the problem of intermittency by supposing a log-normal probability distribution for ϵ. This however, is not the only possibility; indeed a Pandora's box of possibilities is available if the non-uniformity of ϵ is taken into consideration – and the beautiful simplicity of the Kolmogorov (1941*a*) theory, which is its greatest merit, is lost. Anselmet *et al.* (1984) (a paper referred to without dissent in four of the papers in the volume under review) replaces (3), as is customary, by a more general power-law (scaling) relationship

$$\overline{(\Delta u)^p} = A_p r^{\zeta_p}, \tag{4}$$

where the 'structure function exponents' ζ_p are 'determined' for even values of $p \leqslant 18$ by best-fit straight lines on log–log plots. Many current approaches, particularly those involved in multifractal modelling, are concerned with determining the values of ζ_p. Even the relationship (4) however implies some kind of self-similarity throughout the inertial range, a self-similarity that seems incompatible with the phenomenon of concentrated vortex tubes on scales less than the order of the Taylor microscale – a phenomenon widely reported in many direct numerical simulations of turbulence (see, for example, Vincent & Meneguzzi 1991) and also in the experiments of Douady, Couder & Brachet (1991) and Cadot, Douady & Couder (preprint, 1994). Investment of large research effort in the determination of a quantity ζ_p whose very existence is questionable seems potentially misguided. This nevertheless remains a focus for research on the problem of intermittency, various aspects of which are covered in the papers of Kraichnan, Mandelbrot, Frisch, She *et al.*, Sreenivasan, and Hunt & Vassilicos. This volume thus provides an excellent entry to research in this important, if esoteric, corner of the subject.

The paper by Spalding discusses models of k-ϵ type for the determination of the mean properties of turbulent shear flows, i.e. models involving the mean velocity, the kinetic energy density and one supplementary variable. It is not widely known (although recorded by Monin & Yaglom 1971) that Kolmogorov proposed the first model of this kind ('Equations of turbulent motion in an incompressible fluid', Kolmogorov 1942). Spalding himself provides an English translation of this paper, included as an Appendix to his contribution. There are a few troublesome misprints here also: r^a for r^2; $\sum_{i,j}(\partial \bar{v}_i/\partial x_j + \partial \bar{v}_j/\partial x_i)^a$ for $\sum_{i,j}(\partial \bar{v}_i/\partial x_j + \partial \bar{v}_j/\partial x_i)^2$; 'root-mean-square' for 'mean-square'; and most seriously ω for w (both symbols appear in the original Russian version with entirely different meanings); there is a double error in the expression for the internal scale of turbulence, which appears correctly as $\nu^{3/4}(\rho/w)^{1/4}$ in the Russian original and as $\nu^{3/4}(\rho\omega)^{1/4}$ in the translation; despite these flaws, it is valuable to have this translation of a paper which is hard to find in any library even in the Russian original.

Hunt & Vassilicos comment in their paper on the importance of the presentation and elaboration of Kolmogorov's theory by Batchelor (1947). There is no doubt that Batchelor's paper, following on his initial discussion of the theory (and of the related theories of Onsager, von Weizsäcker and Heisenberg) at the VIth International Congress of Theoretical and Applied Mechanics in Paris 1946 (see Batchelor 1946), played a crucial part in the rapid dissemination and validation of Kolmogorov's theory in the Western world after the end of the Second World War.

Kolmogorov's first 1941 paper ranks as "probably the most famous paper ever written on turbulence" (Editor's Preface). It is remarkable in both the clarity and generality of the arguments presented, and, despite the difficulties later revealed by the problem of intermittency (Batchelor & Townsend 1949), it will have a permanent place in the established literature of the subject. This fiftieth anniversary volume serves both to commemorate this work of genius, and also to indicate how far we still have to go to unravel the mysteries locked up within the governing Navier–Stokes equations – to which (in this first paper) Kolmogorov so adroitly avoided making reference!

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